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FACTOR DEMAND THEORY UNDER PERFECT COMPETITION, MONOPOLY, AND M--ETC(U)

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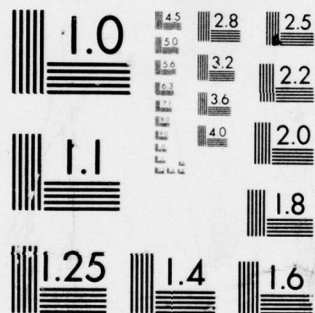
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FACTOR DEMAND THEORY UNDER PERFECT COMPETITION,  
MONOPOLY, AND MONOPSONY

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Surprisingly, most intermediate and advanced microeconomic textbooks fail, in my opinion, to treat factor demand theory adequately. Instead there is a great deal of mystery and some misinformation surrounding the subject. Ferguson [1] in his text Microeconomic Theory lists three (under perfect competition) or four (under monopoly) separate effects of a shift in the price of a factor on the demand for that factor. Despite this, he assures the reader that, when taken together, these effects are such that a rise in price will decrease the demand by an individual firm. No proof is ever attempted. Nicholson [3], on the other hand, identifies only two effects (under perfect competition) and uses "a combination of graphic and pseudomathematical techniques" to suggest why the Giffen Paradox cannot hold for factors. Henderson and Quandt [2] surprisingly identify only one effect, and show using brute force techniques that the demand curve for a factor is always downward sloping.

One reason for this paper, then, is its didactic usefulness, but perhaps more important is to clear up a point of confusion in the theory of derived demand. This problem, recognized in two recent articles by Silberberg[6], [7], arises in the neoclassical theory of the firm under perfect competition because the price of output is assumed to remain fixed in spite of the fact that an increase in the price of any factor will also shift the entire average cost schedule. In the long run, the price of output clearly must change as well to maintain the zero-profit condition, i.e., price equals long-run minimum average cost. In the case of monopoly a shift in the marginal cost schedule changes the firm's optimal output and price, as any undergraduate student of microeconomics knows, so here the confusion is not as apparent.

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This paper contains three parts. First, the factor demand conditions under monopoly are examined because the basic methodology here will be applied to subsequent cases. Second, perfect competition will be examined using the traditional model, and then using the general model; and finally monopsony in some factor markets is also considered. Monopsony conditions, i.e., an upward sloping supply curve to the firm, are faced, for example, when the Department of Defense attempts to enlist individuals for the AVF (All-Volunteer Force). As such this paper should be of interest <sup>FOR THOSE IN</sup> to my Rand colleagues in the military manpower area.

1. Monopoly in the output market; perfect competition in the input markets.

In this section the traditional techniques of comparative statics are used. We seek to prove that the demand curve for a factor is downward sloping. Output  $q$  and output price  $p$  will obviously not remain unchanged but will adjust so that marginal cost and marginal revenue are equal. Let marginal revenue  $MR$  be given by  $d[h(q)q]/dq$  where  $h(q)$  is the demand price.

Factor demand functions are obtained by solving the following (cost-minimization) equations<sup>1</sup>

$$\begin{aligned} f_i - \lambda p_i &= 0 \quad i = 1, 2, \dots, n \\ f(x_1, \dots, x_n) &= q \end{aligned} \quad (1)$$

where  $f$  is the production function with factors  $x_1, \dots, x_n$ ;  $f_i$  is the first partial of  $f$  with respect to  $x_i$ ; and  $p_i$  is the price of  $x_i$ . The effect of an increase in  $p_k$  on the demand for  $x_k$  can be found by differentiating Eqs. (1) totally and solving for  $dx_k/dp_k$ :

$$\begin{bmatrix} 0 & f_1 & f_2 & \dots & f_n \\ -p_1 & f_{11} & f_{12} & \dots & \\ -p_2 & f_{21} & f_{22} & & \\ \vdots & & & \ddots & \\ -p_n & & & & f_{nn} \end{bmatrix} \begin{bmatrix} d\lambda \\ dx_1 \\ \vdots \\ dx_n \end{bmatrix} = \begin{bmatrix} dq \\ \lambda dp_1 \\ \vdots \\ \lambda dp_n \end{bmatrix} \quad (2)$$

<sup>1</sup>Here we are assuming that interior solutions are valid and that the Jacobian of Eqs. (1) is nonzero.

or more simply,

$$\begin{bmatrix} d\lambda \\ dx_1 \\ \vdots \\ dx_n \end{bmatrix} = \begin{bmatrix} dq \\ \lambda dp_1 \\ \vdots \\ \lambda dp_n \end{bmatrix} \quad (2')$$

If we call the first row and column of  $\Gamma$  the zeroth row and column, and designate the cofactor of  $f_{ij}$  as  $\Gamma_{i,j}$ , then from Eq. (2') assuming  $dp_j = 0$  except for  $j = k$ ,

$$dx_k = \frac{\Gamma_{0,k}}{\Gamma} dq + \lambda \frac{\Gamma_{k,k}}{\Gamma} dp_k \quad (3)$$

and

$$d\lambda = \frac{\Gamma_{0,0}}{\Gamma} dq + \lambda \frac{\Gamma_{k,0}}{\Gamma} dp_k \quad (4)$$

Let  $H$  be the traditional bordered Hessian of the production function  $f$

$$H = \begin{bmatrix} 0 & \cdots & f_i \\ \vdots & & \vdots \\ f_j & \cdots & f_{ij} \end{bmatrix} \quad (5)$$

then clearly

$$\left. \begin{aligned} -\lambda \Gamma &= H \\ -\lambda \Gamma_{0,k} &= H_{0,k} \\ -\lambda \Gamma_{k,k} &= H_{k,k} \\ \Gamma_{k,0} &= H_{k,0} \\ \Gamma_{0,0} &= H_{0,0} \end{aligned} \right\} \quad (6)$$

Therefore we can rewrite Eqs. (3) and (4) as

$$dx_k = \frac{H_{0,k}}{H} dq + \lambda \frac{H_{k,k}}{H} dp_k \quad (3')$$

$$d\lambda = -\lambda \frac{H_{0,0}}{H} dq - \lambda^2 \frac{H_{k,0}}{H} dp_k \quad (4')$$

Using the fact that  $\lambda^{-1} = MC = MR$ , the latter condition being required for profit maximization, we have the additional equation

$$d\lambda = -\lambda^2 [qh''(q) + 2h'(q)] dq \quad (7)$$

Equations (3'), (4'), and (7) can also be written in matrix form, which allows for direct computation of all total derivatives:

$$\begin{bmatrix} 0 & 1 & \lambda \frac{H_{0,0}}{H} \\ 1 & 0 & -\frac{H_{0,k}}{H} \\ 0 & 1 & \lambda^2 [qh'' + 2h'] \end{bmatrix} \begin{bmatrix} dx_k \\ d\lambda \\ dq \end{bmatrix} = \begin{bmatrix} -\lambda^2 \frac{H_{k,0}}{H} dp_k \\ \lambda \frac{H_{k,k}}{H} dp_k \\ 0 \end{bmatrix} \quad (8)$$

Hence,

$$\frac{dx_k}{dp_k} = \frac{\lambda \frac{H_{k,k}}{H} \frac{H_{0,0}}{H} - \lambda \frac{H_{0,k}}{H} \frac{H_{k,0}}{H} - \lambda^2 [qh'' + 2h'] \frac{H_{k,k}}{H}}{\frac{H_{0,0}}{H} - \lambda [qh'' + 2h']} \quad (9)$$

Several observations can be made about the signs of the various terms in Eq. (9).  $\frac{H_{k,k}}{H} < 0$ ,  $\frac{H_{0,0}}{H} > 0$ , and  $\frac{H_{0,k}}{H} = \frac{H_{k,0}}{H}$  as a result of the symmetry and negative definiteness of  $H$ . If the marginal revenue curve is falling, then  $[qh'' + 2h'] < 0$ . Hence unambiguously,  $dx_k/dp_k < 0$ . I shall postpone for now an interpretation of each of the terms in Eq. (9) till the next section.

## 2. Perfect competition in both the output and input markets.

The first objective of this section is to prove that the factor demand function is downward sloping under the neoclassical assumption that the price of output remains constant and to interpret the slope as the result of three separate effects.

My rather simple, though to my knowledge unpublished, proof of non-Giffenosity of factors begins with the factor demand function itself. Let  $\psi^k$  be that function for factor  $x_k$

$$\psi^k = \psi^k(p_1, p_2, \dots, p_n, q^*) \quad (10)$$

where  $q^*$  is the firm's optimal output. Differentiating Eq. (10) totally



with respect to  $p_k$  yields the slope of the demand curve.

$$\frac{d\psi^k}{dp_k} = \sum_{j=1}^n \frac{\partial \psi^k}{\partial p_j} \frac{dp_j}{dp_k} \bigg|_{q \text{ constant}} + \frac{\partial \psi^k}{\partial q^*} \frac{dq^*}{dp_k} \quad (11)$$

Letting  $dp_j = 0$  except for  $j=k$ , then

$$\frac{d\psi^k}{dp_k} = \left( \frac{\partial \psi^k}{\partial p_k} \right)_{q \text{ constant}} + \left( \frac{\partial \psi^k}{\partial q^*} \right) \left( \frac{dq^*}{dp_k} \right) \quad (12)$$

The assumption of profit maximization requires

$$p - MC(p_1, \dots, p_n, q^*) = 0 \quad (13)$$

and  $\frac{\partial MC}{\partial q^*} > 0$ . Holding  $p$  constant and differentiating Eq. (13) as an implicit function yields

$$\frac{dq^*}{dp_k} = \frac{- \left( \frac{\partial MC}{\partial p_k} \right)}{\left( \frac{\partial MC}{\partial q^*} \right)} \quad (14)$$

Substituting into Eq. (12), one obtains

$$\frac{d\psi^k}{dp_k} = \left( \frac{\partial \psi^k}{\partial p_k} \right)_{q \text{ constant}} - \left( \frac{\partial \psi^k}{\partial q^*} \right) \left( \frac{\frac{\partial MC}{\partial p_k}}{\frac{\partial MC}{\partial q^*}} \right) \quad (15)$$

The final step requires the use of the duality of the cost function and the factor demand function. The cost function  $C$  has the property that<sup>1</sup>

$$\frac{\partial C(p_1, \dots, p_n, q^*)}{\partial p_k} = \psi^k \quad (16)$$

$$\text{so that } \frac{\partial MC}{\partial p_k} = \frac{\partial^2 C}{\partial p_k \partial q^*} = \frac{\partial \psi^k}{\partial q^*} \quad (17)$$

<sup>1</sup>Incidentally, this provides a simple proof that  $\partial \psi^k / \partial p_j = \partial \psi^j / \partial p_k$  since

$$\frac{\partial \psi^k}{\partial p_j} = \frac{\partial^2 C}{\partial p_k \partial p_j} = \frac{\partial \psi^j}{\partial p_k}.$$

Hence the desired result:

$$\frac{d\psi^k}{dp_k} = \left( \frac{\partial \psi^k}{\partial p_k} \right)_{q \text{ constant}} - \left[ \frac{\left( \frac{\partial \psi^k}{\partial q} \right)^2}{\frac{\partial MC}{\partial q}} \right] \quad (18)$$

The interpretation of Eq. (18) is clear. The first term on the RHS is the pure substitution effect, which by strict quasi-concavity of the production function must be negative. The second term must be positive because as stated before a rising marginal cost curve is the second-order condition for profit maximization. Our theorem is thus proved,  $d\psi^k/dp_k < 0$ .

From Eq. (15) three effects are identifiable: a substitution effect  $\left( \frac{\partial \psi^k}{\partial p_k} \right)_{q \text{ constant}}$  which is always negative; an output effect  $\frac{\partial \psi^k}{\partial q}$ , which is either positive or negative depending on whether the factor is normal or inferior;<sup>1</sup> and a profit-maximizing effect  $\frac{\partial MC}{\partial p_k}$ , which, like the output effect, can be positive or negative.

However, the essential difference between the theory of consumer demand and the theory of factor demand is that, while the consumer must operate within a fixed budget, the firm can vary its output, and by implication its total expenditure on factors. Profit maximization forces the firm to reduce output whenever the marginal cost curve shifts up and to expand output whenever the marginal cost curve shifts down in the vicinity of  $p=MC$ .

The essential link between the profit maximizing effect and the output effect is this: a rise in the price of a normal factor forces the marginal cost curve up, but a rise in the price of an inferior factor forces the marginal cost curve down. Thus in both cases a rise in the price of a factor decreases demand for that factor, assuming that the price of output remains unchanged. An equation similar to Eq. (15) can be used to analyze cross-price effects:

$$\frac{d\psi^k}{dp_j} = \left( \frac{\partial \psi^k}{\partial p_j} \right)_{q \text{ constant}} - \frac{\partial \psi^k}{\partial q} \left( \frac{\frac{\partial MC}{\partial p_j}}{\frac{\partial MC}{\partial q}} \right) \quad (19)$$

<sup>1</sup>The output effect is the equivalent of the income effect in theory of consumer demand.

If both factors are normal and the two factors are complements, i.e., the cross-substitution term is negative, then  $d\psi^k/dp_j$  is unambiguously negative. If the factors are substitutes, i.e., the cross-substitution term is positive, and exactly one of the goods is inferior, then  $d\psi^k/dp_j$  is unambiguously positive. In general, however, the sign of  $d\psi^k/dp_j$  will depend on the magnitudes of the various effects. A complete discussion can be found in Sato and Koizumi [5].

One final remark is useful here before we move on. Factors are called rivals if the second cross partial of the production function,  $f_{ij}$ , is negative, i.e., the marginal product of one factor decreases as more of the second factor is added. Factors are cooperatants (my terminology) if  $f_{ij} > 0$ , which is the usual case. If two factors are rivals, they must be substitutes, but if two factors are cooperatants, they can either be substitutes or complements. Knowing the sign of  $f_{ij}$  is therefore only of limited value.

The entire preceding analysis could have been framed in terms of the first-order conditions for cost minimization as was done for monopoly. The results however can be derived immediately from Eq. (9). Recall that since under the naive model of perfect competition, output price is assumed to be fixed, hence the expression  $[qh'' + 2h']$  is automatically zero. Equation (9) then becomes

$$\frac{dx_k}{dp_k} = \lambda \frac{H_{k,k}}{H} - \lambda \frac{\frac{H_{0,k}}{H} \frac{H_{k,0}}{H}}{\frac{H_{0,0}}{H}} \quad (20)$$

The three effects--substitution, output, and profit-maximizing-- are readily identifiable, and the symmetry of the output and profit-maximizing effects can be seen in the expression

$$\lambda \frac{H_{0,k}}{H} \frac{H_{k,0}}{H}$$

<sup>†</sup>The cross-price effect is given by

$$\frac{dx_k}{dp_j} = \lambda \frac{H_{j,k}}{H} - \lambda \frac{\frac{H_{0,k}}{H} \frac{H_{j,0}}{H}}{\frac{H_{0,0}}{H}}$$

Interchanging the  $k$  and  $j$  indices leaves the RHS unchanged, which proves the well-known result,  $dx_k/dp_j = dx_j/dp_k$ .



The additional term Eq. (9) is what Ferguson calls the "monopoly effect," so it appears then that Ferguson's "count" was in fact justified.

Of course, all of the preceding analysis of perfect competition is wrong, because output price will change. However, the basic framework built in the first section is salvageable. Equations (3') and (4') are still correct but Eq. (7) must be changed. In the long run, the price of output will be equal to the new minimum average cost, which occurs where average and marginal costs are equal. Hence  $\lambda^{-1} = MC = \text{minimum AC}$ . Differentiating this condition,

$$-\lambda^{-2} d\lambda = \sum_{j=1}^n \frac{\partial AC}{\partial p_j} dp_j + \frac{\partial AC}{\partial q} dq \quad (21)$$

Setting  $dp_j = 0$  except for  $j=k$ , and observing that at the minimum point  $\partial AC / \partial q = 0$ , we obtain

$$-\lambda^{-2} d\lambda = \frac{\partial AC}{\partial p_k} dp_k \quad (22)$$

$$= \frac{x_k}{q} dp_k \quad (22')$$

Combining Eqs. (21), (4'), and (22') in matrix notation yields

$$\begin{bmatrix} 0 & 1 & \lambda \frac{H_{0,0}}{H} \\ 1 & 0 & -\frac{H_{0,k}}{H} \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} dx_k \\ d\lambda \\ dq \end{bmatrix} = \begin{bmatrix} -\lambda^2 \frac{H_{k,0}}{H} dp_k \\ \lambda \frac{H_{k,k}}{H} dp_k \\ -\lambda^2 \frac{x_k}{q} dp_k \end{bmatrix}$$

Hence,

$$\frac{dx_k}{dp_k} = \lambda \frac{H_{k,k}}{H} - \lambda \frac{\frac{H_{0,k}}{H} \frac{H_{k,0}}{H}}{\frac{H_{0,0}}{H}} + \lambda \frac{\frac{H_{0,k}}{H} \frac{x_k}{q}}{\frac{H_{0,0}}{H}} \quad (24)$$

Comparing Eqs. (20) and (24) reveals an additional term whose sign, while not necessarily so, will normally be positive.<sup>1</sup>

<sup>1</sup>Solving for the cross-price effect reveals another significant change. It is no longer true in general that reciprocity holds.

A sufficient condition for Eq. (24) to be negative is that the expenditure elasticity of the  $k^{\text{th}}$  factor  $\eta_k(1 - \eta_k) < 0$ .<sup>1</sup>

### 3. Monopoly in the output market; monopsony in some input markets.

Monopsony here is used to indicate that the firm faces a rising supply curve for some inputs. For concreteness, let us assume that the first  $m$  factors are procured competitively and the remaining  $n-m$  factors are procured in monopsonistic markets. Let the supply price of these factors be given by

$$p_i = g^i(x_i, \alpha_i) \quad i = m+1, \dots, n \quad (25)$$

where  $\alpha_i$  is a shift parameter which raises the supply price at any given quantity, e.g.,  $g^i = \delta^i(x_i) + \alpha_i$ . The factor demand functions are obtained by solving<sup>2</sup>

$$\begin{aligned} f_i - \lambda p_i &= 0 & i &= 1, \dots, m \\ f_i - \lambda b_i &= 0 & i &= m+1, \dots, n \\ f(x_1, \dots, x_m, \dots, x_n) &= q \end{aligned} \quad (26)$$

where  $b_i = x_i g^i + g^i$  and  $g^i = \frac{\partial g^i}{\partial x_i}$ ;  $b_i$  is sometimes called the marginal expense of  $x_i$ .

<sup>1</sup>Focusing on the last two terms of Eq. (24), we must only show that

$$\varphi = \frac{H_{0,k}}{H} \left[ \frac{-H_{k,0}}{H} + \frac{x_k}{q} \right] < 0.$$

But by definition  $\eta_k = \frac{q}{x_k} \left( \frac{H_{0,k}}{H} \right) \frac{AC}{MC}$  and at equilibrium  $MC^* = AC^*$ .

Hence  $\varphi = \eta_k(1 - \eta_k) \left( \frac{x_k}{q} \right)^2$ . Since factors are usually classified superior if  $\eta_k > 1$ , normal if  $0 < \eta_k < 1$ , or inferior if  $\eta_k < 0$ , the sufficiency condition seems to demand an extreme condition rather than a normal one.

<sup>2</sup>Maurice and Ferguson [4] investigate the effect of monopsony in factor markets but only the effect of a change in price of a "competitive" factor.



$$\begin{bmatrix} 0 & f_1 & f_2 & \dots & \dots & f_n \\ -p_1 & f_{11} & f_{12} & \dots & \dots & f_{1n} \\ \vdots & \vdots & \vdots & & & \vdots \\ -p_m & f_{m1} & \dots & \dots & \dots & f_{mn} \\ -b_{m+1} & f_{m+1,1} & \dots & f_{m+1,m+1} - \lambda a_{m+1,m+1} & \dots & f_{m+1,n} \\ \vdots & \vdots & & \vdots & & \vdots \\ -b_n & f_{n1} & \dots & \dots & \dots & f_{nn} - \lambda a_{nn} \end{bmatrix} \begin{bmatrix} d\lambda \\ dx_1 \\ \vdots \\ dx_m \\ \vdots \\ dx_n \end{bmatrix} = \begin{bmatrix} dq \\ \lambda dp_1 \\ \vdots \\ \lambda dp_m \\ \lambda \frac{\partial b_{m+1}}{\partial \alpha_{m+1}} d\alpha_{m+1} \\ \vdots \\ \lambda \frac{\partial b_n}{\partial \alpha_n} d\alpha_n \end{bmatrix} \quad (27)$$

where  $a_{ii} = x_i g_{ii}^i + 2g_i^i = m+1, \dots, n$ .

Following the same line of argument as in the previous section, the effect of a price increase in a monopsonistically procured factor on the demand for that factor can be obtained by solving Eqs. (27) for  $dx_k (k=m+1, \dots, n)$  and  $d\lambda$ ; Eq. (7) is now valid again so one can then write:

$$\begin{bmatrix} 0 & 1 & \lambda \frac{F_{0,0}}{F} \\ 1 & 0 & -\frac{F_{0,k}}{F} \\ 0 & 1 & \lambda^2 [qh'' + 2h'] \end{bmatrix} \begin{bmatrix} dx_k \\ d\lambda \\ dq \end{bmatrix} = \begin{bmatrix} -\lambda^2 & \frac{F_{k,0}}{F} & (\frac{\partial b_k}{\partial \alpha_k}) d\alpha_k \\ \lambda \frac{F_{k,k}}{F} & (\frac{\partial b_k}{\partial \alpha_k}) d\alpha_k & \\ 0 & & \end{bmatrix} \quad (28)$$

where  $F$  is the negative definite matrix associated with the second-order condition and  $F_{i,j}$  are the corresponding cofactors of  $F$ .<sup>1</sup>

$${}^1 F = \begin{bmatrix} 0 & \vdots & f_1 \\ \vdots & f_{ii} & f_{ij} \\ f_j & f_{ji} & f_{ii} - \lambda a_{ii} \end{bmatrix}$$

From Eq. (28)

$$\frac{dx_k}{d\alpha_k} = \left( \frac{\partial b_k}{\partial \alpha_k} \right) \left\{ \frac{\lambda \frac{F_{k,k}}{F} \frac{F_{0,0}}{F} - \lambda \frac{F_{0,k}}{F} \frac{F_{k,0}}{F} - \lambda^2 [qh''+2h'] \frac{F_{k,k}}{F}}{\frac{F_{0,0}}{F} - \lambda [qh''+2h']} \right\} \quad (29)$$

The expression inside the brackets is unambiguously negative by the same arguments developed in the previous section;  $(\partial b_k / \partial \alpha_k) > 0$  by our earlier assumption which implies that  $dx_k / d\alpha_k$  is unambiguously negative, that is, an exogeneously leftward shift in the supply curve decreases the quantity of that factor employed.

The effect of a price change in a competitively procured factor on the demand for a monopsonistically procured factor can be investigated by means of Eq. (30).

$$\frac{dx_k}{dp_j} = \frac{\lambda \frac{F_{j,k}}{F} \frac{F_{0,0}}{F} - \lambda \frac{F_{0,k}}{F} \frac{F_{j,0}}{F} - \lambda^2 \frac{F_{j,k}}{F} [qh''+2h']}{\frac{F_{0,0}}{F} - \lambda [qh''+2h']} \quad (30)$$

The sign of the LHS will in general depend on the degree of complementarity or substitutionability between the factors, the magnitude of output effects, and on the demand elasticity. If, however,  $\frac{F_{j,k}}{F} < 0$  that is, the two factors are complements and both factors are normal, then  $dx_k / dp_j$  will be unambiguously negative.

REFERENCES

- [1] Ferguson, C. F., Microeconomic Theory, Richard D. Irwin, Inc., 1969.
- [2] Henderson, James M., and Richard E. Quandt, Microeconomic Theory: A Mathematical Approach, 2nd Edition, McGraw-Hill Book Company, 1971.
- [3] Nicholson, Walter, Microeconomic Theory: Basic Principles and Extensions, The Dryden Press, Inc., 1972.
- [4] Maurice, S. Charles, and C. E. Ferguson, "Factor Demand Elasticity under Monopoly and Monopsony," Economica, Vol. 40, May 1973, pp. 180-186.
- [5] Sato, Ryuzo, and T. Koizumi, "Substitutability, Complementarity, and the Theory of Demand," Review of Economic Studies, Vol. 37, 1970, pp. 107-118.
- [6] Silberberg, Eugene, "A Revision of Comparative Statics Methodology in Economics, or How To Do Comparative Statics on the Back of an Envelope," Journal of Economic Theory, Vol. 7, No. 2, February 1974, pp. 159-172.
- [7] \_\_\_\_\_, "The Theory of the Firm in 'Long-Run' Equilibrium," American Economic Review, Vol. LXIV, No. 4, September 1974, pp. 734-741.